1. Relations and Functions

1 mark Questions

1. If $n(A \times B) = 6$ and $A = \{1,3\}$ then $n(B)$ is SEP-21
   (A) 1 (B) 2 **(C) 3** (D) 6
2. $A = \{a, b, p\}, B = \{2,3\}, C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is PTA-3
   (A) 8 (B) 20 **(C) 12** (D) 16
3. If $A = \{1,2\}, B = \{1,2,3,4\}, C = \{5,6\}$ and $D = \{5,6,7,8\}$ then state which of the following statement is true SEP-20
   (A) $(\boldsymbol{A \times C}) \subset (\boldsymbol{B \times D})$ (B) $(B \times D) \subset (A \times C)$
   (C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$
4. If there are 1024 relations from a set $A = \{1,2,3,4,5\}$ to a set B, then the number of element in B is
   (A) 3 **(B) 2** (C) 4 (D) 8 PTA-2, JUL-22
5. The range of the relations $R = \{(x, x^2) | x$ is a prime number less than 13$\}$ is PTA-4, JUL-22
   (A)$\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$ **(C) $\{4,9,25,49,121\}$** (D) $\{1,4,9,25,49,121\}$
6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is PTA-6, MAY-22
   (A) $(2, -2)$ (B) $(5,1)$ (C) $(2,3)$ **(D) $(3, -2)$**
7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is
   (A) m^n (B) n^m **(C) $2^{mn} - 1$** (D) 2^{mn} PTA-1
8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b respectively.
   (A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)
9. Let $A = \{1,2,3,4\}$ and $B = \{4,8,9,10\}$. A function $f: A \to B$ given by $f = \{(1,4), (2,8), (3,9), (4,10)\}$ is a
   (A) Many-one function (B) Identity function PTA-4
   (C) One-to-one function (D) Into function
10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
    (A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ **(C) $\frac{2}{9x^2}$** (D) $\frac{1}{6x^2}$
11. If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to PTA-2
    (A) 7 (B) 49 (C) 1 (D) 14
12. Let f and g be two functions given by
    $f = \{(0,1), (2,0), (3, -4), (4,2), (5,7)\}$
    $g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ then the range of $f \circ g$ is
    (A) $\{0,2,3,4,5\}$ (B) $\{-4,1,0,2,7\}$ (C) $\{1,2,3,4,5\}$ **(D) $\{0, 1, 2\}$**
13. Let $f(x) = \sqrt{1 + x^2}$ then
    (A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
    (C) $f(xy) \leq f(x).f(y)$ (D) None of these
14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
    (A) $(-1,2)$ **(B) $(2, -1)$** (C) $(-1, -2)$ (D) $(1,2)$ PTA-6
15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is
    (A)linear (B) cubic (C) reciprocal **(D) quadratic** PTA-5

2 mark Questions

1. **Find $A \times B, A \times A$ and $B \times A$ (iii) $A = \{m, n\}; B = \emptyset$** PTA-1

(iii) $A = \{m, n\}; B = \emptyset$

$A \times B = \{\ \ \}$

$A \times A = \{m, n\} \times \{m, n\}$

$= \{(\boldsymbol{m}, \boldsymbol{m}), (\boldsymbol{m}, \boldsymbol{n}), (\boldsymbol{n}, \boldsymbol{m}), (\boldsymbol{n}, \boldsymbol{n})\}$

$B \times A = \{\ \ \}$

2. **Let $A = \{1, 2, 3\}$ and $B = \{x | x$ is a prime number less than 10$\}$. Find $A \times B$ and $B \times A$.** MAY-22

$A = \{1,2,3\}$

$B = \{x | x$ is a prime number less than $10\}$

$= \{2,3,5,7\}$

$A \times B = \{1,2,3\} \times \{2,3,5,7\}$

$= \{(\boldsymbol{1}, \boldsymbol{2}), (\boldsymbol{1}, \boldsymbol{3}), (\boldsymbol{1}, \boldsymbol{5}), (\boldsymbol{1}, \boldsymbol{7}), (\boldsymbol{2}, \boldsymbol{2}), (\boldsymbol{2}, \boldsymbol{3}),$
$(\boldsymbol{2}, \boldsymbol{5}), (\boldsymbol{2}, \boldsymbol{7}), (\boldsymbol{3}, \boldsymbol{2}), (\boldsymbol{3}, \boldsymbol{3}), (\boldsymbol{3}, \boldsymbol{5}), (\boldsymbol{3}, \boldsymbol{7})\}$

$B \times A = \{2,3,5,7\} \times \{1,2,3\}$

$= \{(\boldsymbol{2}, \boldsymbol{1}), (\boldsymbol{2}, \boldsymbol{2}), (\boldsymbol{2}, \boldsymbol{3}), (\boldsymbol{3}, \boldsymbol{1}), (\boldsymbol{3}, \boldsymbol{2}), (\boldsymbol{3}, \boldsymbol{3}), (\boldsymbol{5}, \boldsymbol{1}), (\boldsymbol{5}, \boldsymbol{2}), (\boldsymbol{5}, \boldsymbol{3}), (\boldsymbol{7}, \boldsymbol{1}), (\boldsymbol{7}, \boldsymbol{2}), (\boldsymbol{7}, \boldsymbol{3})\}$

3. **Let $A = \{1, 2, 3, 4, \dots, 45\}$ and R be the relation defined as "is square of a number" on A.Write R as a subset of $A \times A$. Also, find the domain and range of R.** SEP-21

Given $A = \{1,2,3,4, \dots, 45\}$

$A \times A = \{(1,1), (1,2), (1,3), (1,4) \dots \dots (45,45)\}$

Then, R be the relation defined as is "square of a number" on A.

Hence, $R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$

So $R \subseteq A \times A$

The domain of $R = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \boldsymbol{4}, \boldsymbol{5}, \boldsymbol{6}\}$

The range of $R = \{\boldsymbol{1}, \boldsymbol{4}, \boldsymbol{9}, \boldsymbol{16}, \boldsymbol{25}, \boldsymbol{36}\}$

4. **A Relation R is given by the set $\{(x, y)/y = x + 3,\ x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range (PTA-5)**

$R = \{(x, y)/y = x + 3, x \in \{0,1,2,3,4,5\}\}$

Here domain $(x) = \{0,1,2,3,4,5\}$

Co-domain $(y) = x + 3$

$y_0 = 0 + 3 = 3$, $\qquad y_3 = 3 + 3 = 6$

$y_1 = 1 + 3 = 4$, $\qquad y_4 = 4 + 3 = 7$

$y_2 = 2 + 3 = 5$, $\qquad y_5 = 5 + 3 = 8$

$R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain $= \{\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \boldsymbol{4}, \boldsymbol{5}\}$

Range $= \{\boldsymbol{3}, \boldsymbol{4}, \boldsymbol{5}, \boldsymbol{6}, \boldsymbol{7}, \boldsymbol{8}\}$

5. **Show that the function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one – one function** SEP-20

The function $f: \mathbb{N} \to \mathbb{N}$ defined by

$f(m) = m^2 + m + 3$

$m = 1, f(1) = (1)^2 + 1 + 3 = 1 + 1 + 3 = 5$

$m = 2, f(2) = (2)^2 + 2 + 3 = 4 + 2 + 3 = 9$

$m = 3, f(3) = (3)^2 + 3 + 3 = 9 + 3 + 3 = 15$

$m = 4, f(4) = (4)^2 + 4 + 3 = 16 + 4 + 3 = 23$

Since different elements of N have different images in the codomain the function of $\boldsymbol{f}$ **is one-one function.**

6. **Write the domain of the following real functions** PTA-6

i) $\boldsymbol{f(x) = \frac{2x+1}{x-9}}$ **iii)** $\boldsymbol{g(x) = \sqrt{x-2}}$

i) $f(x) = \frac{2x+1}{x-9}$

If $x = 9$ then $f(-9)$ is not defined

Hence f is defined for all real numbers except at $x = 9$.

So domain of $f = R - \{9\}$

iii) $g(x) = \sqrt{x-2}$

If $x \in (-\infty, 2)$ $g(x)$ is not real

If $x \in [2, \infty)$ $g(x)$ is real

$\therefore$ the Domain is $[2, \infty)$

5 mark Questions

1. **Let $A = \{x \in \mathbb{W} | x < 2\}, B = \{x \in \mathbb{N} | 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that**

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ SEP-21, PTA-5

LHS: $B \cap C = \{2,3,4\} \cap \{3,5\} = \{3\}$

$A \times (B \cap C) = \{0,1\} \times \{3\}$

$= \{(\mathbf{0}, \mathbf{3}), (\mathbf{1}, \mathbf{3})\}$ ....(1)

RHS: $A \times B = \{0,1\} \times \{2,3,4\} = \{(\mathbf{0},\mathbf{2}), (\mathbf{0},\mathbf{3}), (\mathbf{0},\mathbf{4}), (\mathbf{1},\mathbf{2}), (\mathbf{1},\mathbf{3}), (\mathbf{1},\mathbf{4})\}$

$A \times C = \{0,1\} \times \{3,5\} = \{(0,3), (0,5), (1,3), (1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,2), (\mathbf{0},\mathbf{3}), (0,4), (1,2), (\mathbf{1},\mathbf{3}), (1,4)\} \cap \{(\mathbf{0},\mathbf{3}), (0,5), (\mathbf{1},\mathbf{3}), (1,5)\}$

$= \{(\mathbf{0},\mathbf{3}), (\mathbf{1},\mathbf{3})\}$ ....(2)

From (1) and (2),

$$\boldsymbol{A \times (B \cap C) = (A \times B) \cap (A \times C)}$$

2. If $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$, show that $A \times A = (B \times B) \cap (C \times C)$. **(JUL-22)**

$A \times A = (B \times B) \cap (C \times C)$

LHS: $A \times A = \{5,6\} \times \{5,6\}$

$= \{(\mathbf{5}, \mathbf{5}), (\mathbf{5}, \mathbf{6}), (\mathbf{6}, \mathbf{5}), (\mathbf{6}, \mathbf{6})\}$ ……….(1)

RHS:

$B \times B = \{4,5,6\} \times \{4,5,6\}$

$= \{(4,4), (4,5), (4,6), (5,4), (\mathbf{5}, \mathbf{5}),$

$(\mathbf{5}, \mathbf{6}), (6,4), (\mathbf{6}, \mathbf{5}), (\mathbf{6}, \mathbf{6})\}$

$C \times C = \{5,6,7\} \times \{5,6,7\} = \{(\mathbf{5}, \mathbf{5}), (\mathbf{5}, \mathbf{6}), (5,7), (\mathbf{6}, \mathbf{5}), (\mathbf{6}, \mathbf{6}), (6,7), (7,5), (7,6), (7,7)\}$

$(B \times B) \cap (C \times C) = \{(\mathbf{5}, \mathbf{5}), (\mathbf{5}, \mathbf{6}), (\mathbf{6}, \mathbf{5}), (\mathbf{6}, \mathbf{6})\}$……….(2)

From (1) and (2), $\boldsymbol{A \times A = (B \times B) \cap (C \times C)}$

3. Let $A = \{x \in \mathbb{W} | x < 2\}, B = \{x \in \mathbb{N} | 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **(PTA-2)**

$A = \{x \in \mathbb{W} | x < 2\} = \{0,1\}$, $B = \{x \in \mathbb{N} | 1 < x \leq 4\} = \{2,3,4\}$, $C = \{3,5\}$

LHS:

$B \cup C = \{2,3,4\} \cup \{3,5\} = \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$= \{(\mathbf{0}, \mathbf{2}), (\mathbf{0}, \mathbf{3}), (\mathbf{0}, \mathbf{4}), (\mathbf{0}, \mathbf{5}), (\mathbf{1}, \mathbf{2}), (\mathbf{1}, \mathbf{3}), (\mathbf{1}, \mathbf{4}), (\mathbf{1}, \mathbf{5})\}$ ……..(1)

RHS:

$A \times B = \{0,1\} \times \{2,3,4\} = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{0,1\} \times \{3,5\} = \{(0,3), (0,5), (1,3), (1,5)\}$

$(A \times B) \cup (A \times C) = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\} \cup \{(0,3), (0,5), (1,3), (1,5)\}$

$= \{(\mathbf{0}, \mathbf{2}), (\mathbf{0}, \mathbf{3}), (\mathbf{0}, \mathbf{4}), (\mathbf{0}, \mathbf{5}), (\mathbf{1}, \mathbf{2}), (\mathbf{1}, \mathbf{3}), (\mathbf{1}, \mathbf{4}), (\mathbf{1}, \mathbf{5})\}$ ……….(2)

From (1) and (2), $\boldsymbol{A \times (B \cup C) = (A \times B) \cup (A \times C)}$

4. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number, Verify that (i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **(SEP-20)**

A = The set of all natural numbers less than 8 = $\{1,2,3,4,5,6,7\}$

B = The set of all prime numbers less than 8 = $\{2,3,5,7\}$

C = The set of even prime number = $\{2\}$

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS: $A \cap B = \{1,2,3,4,5,6,7\} \cap \{2,3,5,7\}$

$= \{2,3,5,7\}$

$(A \cap B) \times C = \{2,3,5,7\} \times \{2\} = \{(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{2}),(\mathbf{5},\mathbf{2}),(\mathbf{7},\mathbf{2})\}$ ....(1)

RHS:

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{2,3,5,7\} \times \{2\} = \{(2,2),(3,2),(5,2),(7,2)\}$

$(A \times C) \cap (B \times C) = \{(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{2}),(\mathbf{5},\mathbf{2}),(\mathbf{7},\mathbf{2})\}$ ....(2)

From (1) and (2), $(\boldsymbol{A \cap B}) \times \boldsymbol{C} = (\boldsymbol{A \times C}) \cap (\boldsymbol{B \times C})$

5. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number, Verify that

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ MAY-22

LHS: $B - C = \{\cancel{2},3,5,7\} - \{\cancel{2}\} = \{3,5,7\}$

$A \times (B - C) = \{1,2,3,4,5,6,7\} \times \{3,5,7\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),$
$(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$............(1)

RHS: $A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),$
$(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),(7,2),(7,3),(7,5),(7,7)\}$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\} = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C)$

$= \{(\cancel{1,2}),(1,3),(1,5),(1,7),(\cancel{2,2}),(2,3),(2,5),(2,7),(\cancel{3,2}),(3,3),(3,5),(3,7),(\cancel{4,2}),(4,3),$
$(4,5),(4,7),(\cancel{5,2}),(5,3),(5,5),(5,7),(\cancel{6,2}),(6,3),(6,5),(6,7),(\cancel{7,2}),(7,3),(7,5),(7,7)\}$
$- \{(\cancel{1,2}),(\cancel{2,2}),(\cancel{3,2}),(\cancel{4,2}),(\cancel{5,2}),(\cancel{6,2}),(\cancel{7,2})\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),$
$(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$............(2)

From (1) and (2), $\boldsymbol{A} \times (\boldsymbol{B - C}) = (\boldsymbol{A \times B}) - (\boldsymbol{A \times C})$

6. **Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible. (ii) $\{(x, y)|y = x + 3, x, y$ are natural numbers $<10\}$** **(JUL-22)**

(ii) $\{(x, y)|y = x + 3, x, y$ are natural numbers $<10\}$

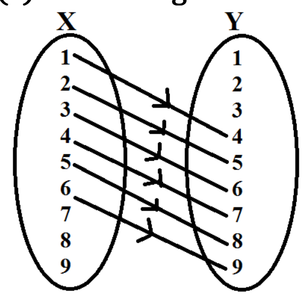
Given, x, y are natural numbers < 10

$X = \{1,2,3,4,5,6,7,8,9\}$, $y = x + 3$

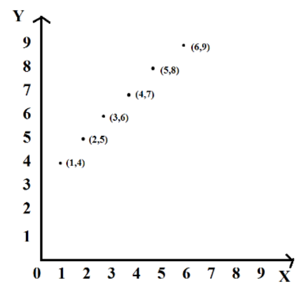
Here $y_1 = 4,\quad y_2 = 5,\quad y_3 = 6,$

$y_4 = 7,\quad y_5 = 8,\quad y_6 = 9$

(a) Arrow diagram



(b) graph



(c) Roster Form

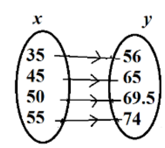
$R = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

7. **The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants. (i) Check if this relation is a function. (ii) Find a and b(iii) Find the height of a person whose forehand length is 40cm (iv) Find the length of forehand of a person if her height is 53.3 inches.** PTA-4

Length x of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

Given $y = ax + b$

(i) Arrow diagram



Each element in x is associated with a unique element in y

Yes, this relation is a function

(ii) find a and b

From the table

$35a + b = 56$ ……………(1)

$45a + b = 65$ …………..(2)

(−) (−) (−)

$-10a = -9$

$a = \frac{9}{10} = 0.9$

$a = 0.9$ substitute in (1)

$35(0.9) + b = 56$

$31.5 + b = 56$

$b = 56 - 31.5 = 24.5$

$\boldsymbol{a = 0.9}$ **and** $\boldsymbol{b = 24.5}$

(iii) Length $= 40$cm, $a = 0.9, b = 24.5$

$y = ax + b$

$= (0.9)(40) + 24.5 \quad = 60.5$

The height of a person whose forehand length is 40 cm **= 60.5 inches.**

(iv) Height $= 53.3$ inches

$y = ax + b$

$53.3 = (0.9)x + 24.5 \quad = 0.9x + 24.5$

$53.3 - 24.5 = 0.9x$

$28.8 = 0.9x$

$x = \frac{28.8}{0.9} = 32 \Rightarrow x = 32$ cm

The length of forehand of a person = **32 cm**

8. **A function $f:[-5,9]\to\mathbb{R}$ is defined as follows: $f(x)=\begin{cases}6x+1; & -5\le x<2\\ 5x^2-1; & 2\le x<6\\ 3x-4; & 6\le x\le 9\end{cases}$** PTA-4

Find (ii) $f(7)-f(1)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$$f(x)=\begin{cases}6x+1; & -5\le x<2 & ;\text{ Where } x=-5,-4,-3,-2,-1,0,1\\ 5x^2-1; & 2\le x<6 & ;\text{ Where } x=2,3,4,5\\ 3x-4; & 6\le x\le 9 & ;\text{ Where } x=6,7,8,9\end{cases}$$

(ii) $f(7)-f(1)$ When $x=7$ $f(x)=3x-4$ $f(7)=3(7)-4=21-4=17$ When $x=1$ $f(x)=6x+1$ $f(1)=6(1)+1=6+1=7$ $\therefore f(7)-f(1)=17-7=\mathbf{10}$	(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$ When $x=-2,\ f(x)=6x+1$ $f(-2)=6(-2)+1=-12+1=-11$ When $x=6, f(x)=3x-4$ $f(6)=3(6)-4=18-4=14$ When $x=4,\ f(x)=5x^2-1$ $f(4)=5(4)^2-1=80-1=79$ $\frac{2f(-2)-f(6)}{f(4)+f(-2)}=\frac{2(-11)-14}{79+(-11)}=\frac{-22-14}{79-11}=\frac{-36}{68}=-\mathbf{\frac{9}{17}}$

9. **The distance S an object travels under the influence of gravity in the time t seconds is given by $S(t)=\frac{1}{2}gt^2+at+b$ where, (g is the acceleration due to gravity), a,b are constants. Verify whether the function $S(t)$ is one-one or not.** PTA-3

Given $S(t)=\frac{1}{2}gt^2+at+b$ (a,b constants)

Now take $t=1,2,3,\ldots$ seconds

$t=1,\quad S(1)=\frac{1}{2}g(1)^2+a(1)+b$

$=\frac{1}{2}g+a+b=\mathbf{0.5g+a+b}$

$t=2,\quad S(2)=\frac{1}{2}g(2)^2+a(2)+b$

$=\mathbf{2g+2a+b}$

$t=3,\quad S(3)=\frac{1}{2}g(3)^2+a(3)+b$

$=\mathbf{4.5g+3a+b}$

Since distinct elements of A have distinct image in B.

Yes, it is an one-one function.

10. The function $'t'$ which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find PTA-1

(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C) = 212$

(v) the temperature when the Celsius value is equal to the Fahrenheit value

The function t is defined by, $t(C) = F$, where $F = \frac{9}{5}C + 32$

(i) $t(0) = \frac{9}{5}(0) + 32 = \mathbf{32°F}$

(ii) $t(28) = \frac{9}{5}(28) + 32$

$= 9(5.6) + 32$

$= 50.4 + 32$

$= \mathbf{82.4°F}$

(iii) $t(-10) = \frac{9}{5}(-10) + 32$

$= -18 + 32$

$= \mathbf{14°F}$

(iv) When $t(C) = 212$

$\frac{9}{5}C + 32 = 212$

$\frac{9}{5}C = 212 - 32 = 180$

$C = \frac{180\times5}{9} = \mathbf{100°C}$

(v) we know that

$t(C) = F$ where $F = \frac{9}{5}C + 32$

$t(F) = C$ where $C = \frac{9}{5}F + 32$

If the temperatures are same then two $'t'$s in the formula should represent the same temperature. So then we multiply each side by $\left(-\frac{5}{4}\right)$

$t = \frac{9}{5}t + 32°$

$t - \frac{9}{5}t = 32°$

Multiply each side by $\left(-\frac{5}{4}\right)$

$-\frac{5}{4}\left(t - \frac{9}{5}t\right) = 32° \times \left(-\frac{5}{4}\right)$

$-\frac{5}{4}t + \frac{9}{4}t = -40°$

$\frac{-5t+9t}{4} = -40°$

$\frac{4t}{4} = -40°$

$\boldsymbol{t = -40°}$

11. If $f(x) = x^2 - 1, g(x) = x - 2$ find a, if $g \circ f(a) = 1$ PTA-2

Given $f(x) = x^2 - 1, g(x) = x - 2$

$g \circ f(x) = g(f(x)) = g(x^2 - 1)$

$= x^2 - 1 - 2$

$= x^2 - 3$

Given $g \circ f(a) = 1$

Hence $a^2 - 3 = 1$

$a^2 = 1 + 3$

$a^2 = 4$

$\boldsymbol{a = \pm 2}$

12. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?

PTA-6

$f: R \to R$ defined by $f(x) = x^5$

$f \circ f(x) = f(f(x))$

$= f(x^5)$

$= (x^5)^5 = x^{25}$

$f \circ f(1) = (1)^{25} = 1$

$f \circ f(2) = (2)^{25}$

$f \circ f(3) = (3)^{25}$

Since each elements in f have distinct images, f is one-one

$g: R \to R$ defined by $g(x) = x^4$

$g \circ g(x) = g(g(x)) = g(x^4)$

$= (x^4)^4$

$= x^{16}$

$g \circ g(-1) = (-1)^{16} = 1$

$g \circ g(1) = (1)^{16} = 1$

$g \circ g(2) = (2)^{16}$

Thus two distinct elements -1

and 1 have same images.

Hence g is not one-one

$f \circ g(x) = f(g(x))$

$= f(x^4)$

$= (x^4)^5 = x^{20}$

$f \circ g(1) = (1)^{20} = 1$

$f \circ g(-1) = (-1)^{20} = 1$

Thus two distinct elements -1 and 1 have same images. Hence **$f \circ g$ is not one-one.**

13. Consider the functions $f(x), g(x), h(x)$ as given below, show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.

(iii) $f(x) = x - 4, g(x) = x^2$ and $h(x) = 3x - 5$

PTA-2

$f \circ g(x) = f(g(x))$

$= f(x^2) = x^2 - 4$

Then $(f \circ g) \circ h(x) = f \circ g(h(x))$

$= f \circ g(3x - 5)$

$= (3x - 5)^2 - 4$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$……..(1)

$(g \circ h)x = g(h(x))$

$= g(3x - 5) = (3x - 5)^2$

$= 9x^2 - 30x + 25$

$f \circ (g \circ h)(x) = f(9x^2 - 30x + 25)$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$ ………..(2)

From (1) and (2), $(\boldsymbol{f} \circ \boldsymbol{g}) \circ \boldsymbol{h} = \boldsymbol{f} \circ (\boldsymbol{g} \circ \boldsymbol{h})$